

Clearly, an investigation of a single fire provides only a small fraction of all the evidence that must be accumulated before thoroughly dependable conclusions can be drawn. Other fires, under similar conditions of temperature, humidity, and wind, might exhibit entirely different behavior if the fuels, the topography, or the

tactics of suppression differed materially. Several of these investigations must be made in each distinct timber type under different weather conditions before the major variations of fire behavior can be accurately related to the behavior of the weather.

## INVESTIGATION OF RAINFALL PERIODICITIES BETWEEN 1 1/6 AND 2 1/2 YEARS BY USE OF SCHUSTER'S PERIODOGRAM

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### SYNOPSIS

The present paper continues previous applications of Schuster's periodogram to the rainfall of the world, in this case for shorter periods than considered in them. The evidence continues favorable, and is even stronger than before, in favor both of the existence of what may be termed a "spectrum" of related periods and of their relation to the sun-spot period. It also begins to differentiate strongly between fairly constant periods and variable cycles, in favor of the former.

This paper and a short investigation of economic values of statistical examinations of rainfall periodicities to be published in the next issue of this REVIEW, complete a series of studies (1) of the rainfall of the world, begun about seven years ago. In these it was concluded definitely that periods or cycles do exist, but whether of constant length and amplitude remained uncertain.

The last few papers have been an application of Schuster's periodogram to these data and a study of the method itself. In each paper a stretch of shorter periods than the preceding ones has been considered.

Since Schuster's (2) original papers are available and also the preceding papers of this series which considered the method in detail, it is unnecessary to explain the method.

Given data  $q_1, \dots, q_n$ , assume any period  $P$ , times the datum interval. Let  $\varphi_i$  be the phase angle for the datum  $q_i$  so that

$$\varphi_{i+1} - \varphi_i \equiv \frac{2\pi}{P}$$

$$\varphi_1 \equiv 0$$

$$A_j \equiv \sum_{i=1}^n q_i \cos \varphi_i$$

$$B_j \equiv \sum_{i=1}^n q_i \sin \varphi_i$$

$$I_j \equiv \frac{A_j^2 + B_j^2}{n}$$

and is proportional to the square of the amplitude of the best sine curve of period  $P_j$  to fit the data.

$$\tan \Phi_j = \frac{B_j}{A_j}; I_{mean} = \frac{\sum \sigma_i^2}{2(n-1)}$$

Where  $\sigma_i$  is the deviation of  $q_i$  from the mean  $q$ .

$$H_j \equiv \frac{I_j}{I_{mean}}; I'_j = I_j \frac{(x-y)^2}{4 \sin^2 \frac{1}{2}(x-y)}$$

where  $X$  is the phase of  $q_i$  and  $Y$  that of  $q_{i+1}$  in radian measure.  $I'$  is the value of  $I$  which would have been secured had much shorter datum intervals been used.

$I$  not  $I'$  must be used in computing the probability of securing any given  $I$  by accident.

In this paper semiyearly rainfall means from the Pacific coast of the United States, from the Punjab and from the British Isles are used. These data, in the form of percentage departures from normal, are given as Table 1. The periods computed overlap those of the preceding paper. First, periodograms for periods between  $1\frac{1}{6}$  and  $2\frac{1}{2}$  years are computed from each half of the data of each of these sections. These chronologically independent periodograms are then compared. Finally a periodogram is computed from the total data of each section and conclusions drawn.

Before comparing these chronologically different periodograms a few points respecting the evidence given by periodograms may be considered.

(a) The fact that the peaks of chronologically different periodograms are of different intensity or of slightly different position is often considered as indicating variability of period. On page 480 of the October, 1924, MONTHLY WEATHER REVIEW are two very different appearing periodograms, made from different long stretches of data, composed by the addition of two sine curves of equal amplitude. When it is remembered that in these data there were only two periods and no accidental errors at all, the possibilities of variation, where the data may be composed of a whole spectrum of periods plus large accidental errors, are seen to be great enough almost to preclude evidence in favor of a variable cycle unless a very great number of such cycles have been completed.

(b) The comparison of periodograms by Pearson's correlation coefficient, while useful, minimizes the relationship between them for the following reasons:

1. Suppose that in the periodograms one real peak is shown equally in both. This one peak will give a positive correlation but all the rest will give a zero correlation, since the theory of the periodogram shows that it varies under the accidental error law, except in the vicinity of real periods. We would, therefore, in this case, expect from the periodograms a small positive coefficient, comparable with the probable error, and telling us, therefore, little or nothing. *A large positive correlation becomes strong evidence, consequently, in favor of a whole spectrum of real periods even where the separate peaks may be too low in height to carry much weight individually.*

2. If there are variable cycles, or if the interference of periods, or the accidental errors, have produced slight shifts in the positions of peaks, the two periodograms will give a negative coefficient, sometimes very large. This is beautifully illustrated in the periodograms of the Pacific coast. At about 2.30 years a high peak starts in each. At about 2.33 years the one from the early data reaches its maximum  $H=4.8$ , here exactly coinciding with the later periodogram. This later one continues to rise to  $H=10.5$ , while the earlier one falls rapidly. One half indicates a period at  $P=2.33$  years, the other at  $P=2.44$ ,

each within the accidental range of their mean, yet they give such large negative terms to what is otherwise a rather large positive correlation as to reduce it practically to zero.

3. In computing the correlation coefficient the use of the computed  $I_m$  magnifies the apparent relationship, and the use of the measured just as obviously decreases it. In results given in this paper the measured  $I_m$  has always been used.

In forming the six month means for use in this paper, the same monthly means of the Pacific coast and of the

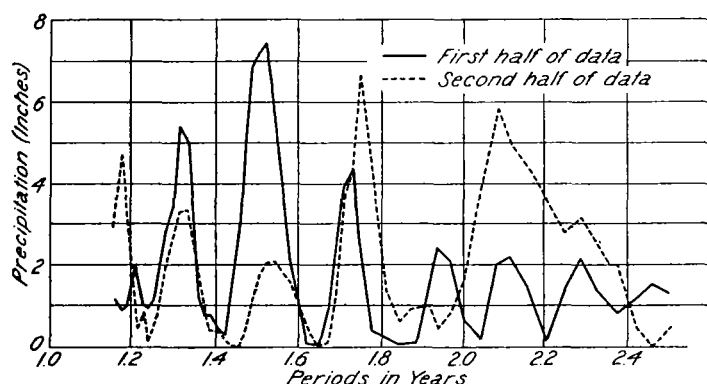


FIG. 1.—Rainfall periodogram of the Punjab

Punjab were used as in the last paper. For the British Isles additional data were received through the United States Weather Bureau and the monthly percentages of normal of Armagh, Chilgrove, Edinburgh, Greenwich, and Stonyhurst were averaged.<sup>1</sup>

We shall discuss the data in the order of definiteness of the results, taking the Punjab, which is least definite, first. This is to be expected, for while the same number of years are used in the earlier periodogram as for other sections, the latter, because of the shorter stretch, has about two dozen less from which to compute the periodogram. The three periodograms from the entire stretch of data will be included in one discussion after that of all those from the halves of the data have been completed. We find that now when we have shorter periods, and, therefore, a greater number of cycles, the values of  $H$  are greater than in the last paper, the highest being 7.48 and 5.40 in the first half of the data and 6.44 and 5.77 in the second half. In the last paper the highest peaks were  $H=4.26$  and 3.41. There the values were entirely inconclusive; here they are quite definite in their indication of reality. The accidental expectancy ratio of  $H=7.48$  is one in 2,100. If our data followed the accidental laws the  $H$ 's should be of about the same height regardless of the number of cycles completed. If there be a spectrum of real periodicities between long and short periods we should find just as we have, that  $H$  increases with the number of cycles.

The periodograms are plotted as Figure 1, which brings out their similarities and divergencies. We see that except at the extreme ends of the periodograms there is an almost perfect agreement in the positions of the peaks of these chronologically independent periodograms. Remembering the limitations to the application of the correlation coefficient we shall nevertheless compute it and find  $r = +0.167$ ;  $\epsilon_r = 0.0868$ ;  $\frac{r}{\epsilon_r} = 1.925$ . Such

a correlation, although somewhat favorable, is not at all definite, since one pair out of every 47 pairs of curves should give a relationship of this size by mere accident. However, a glance at the periodograms shows that this low coefficient has come, not from the usual error distribution of dissimilarities but from the ends of the curves, especially the end of longer periods. Here the later curve shows a high peak with a slight side protuberance, while the early curve shows, in exactly the same place, two low peaks. These are twice as high as the computed mean and would, therefore, give a positive correlation if we had used it instead of the measured mean. However, on account of the high peaks appearing in the periodograms, they are lower than that measured and for this reason give in combination with the high peak a very large negative correlation. If we omit these ends, using all the points 16–54, we find that these 39 central points give  $r = +0.339$  which is 3.55 times its probable error. The accidental expectancy ratio of such a correlation is one in 1,200, indicating a correlation far higher than we would expect from mere accident. The indications are, therefore, that several periods or cycles did persist throughout both halves of the data with a constancy greater than we could expect through accident.

When we turn to the Pacific coast periodograms we find a very similar situation, although here the lower peak has capsized after starting up with the higher one. Here, on account of the fact that the high peak is truly remarkable, being 10.50 times the computed mean, the negative factors introduced lower the correlation, although it is still slightly positive, to less than its probable error. Once more we must notice that the use of the computed mean height would have given a positive correlation between these peaks. Using the same 39 central points which we used for The Punjab, we obtain  $r = +0.435$ ,  $\epsilon_r = 0.0876$  and  $\frac{r}{\epsilon_r} = 4.96$ . The accidental expectancy

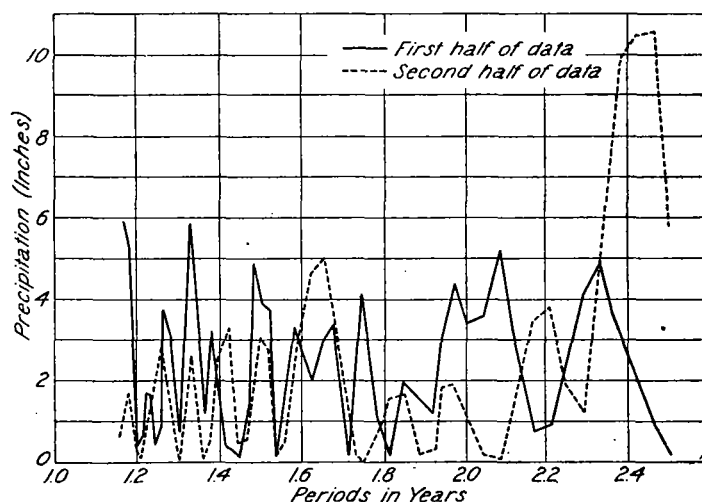


FIG. 2.—Rainfall periodogram of the U. S. Pacific coast

ratio of such a correlation is one in 20,300, a quite definite proof of relationship. Entirely apart from the mathematical proof given by these correlations, a mere visual examination shows quite well the persistancy of the spectrum of periods.

In the case of the periodograms of the British Isles a glance is sufficient to show that the relationship can not be accidental. Throughout the length of the curves there is scarcely a disagreement. Out of ten peaks, nine show almost perfect agreement. None are high but

<sup>1</sup> These data are British Isles A. of Table 1. Since reading this paper at Philadelphia, still more data have been received. These lengthen the records and give Rothsay as an additional station. The new means, not used in this paper, are given as British Isles B. They are being used in a numerical example to demonstrate the value of a new form of periodogram.

we can not doubt at all their reality and approximate constancy. We find  $r = +0.432$ ;  $\epsilon_r = 0.0726$ ;  $\frac{r}{\epsilon_r} = 5.96$ . This ratio tells us in exact numbers what inspection has already told us, i. e., that the peaks are real. The expectancy ratio is one in 150,200.

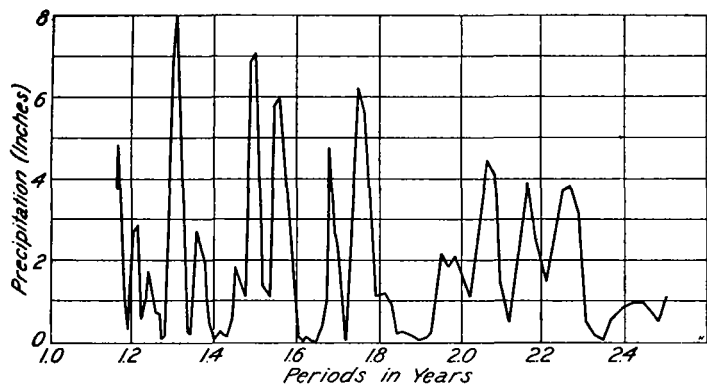


FIG. 3.—Rainfall periodogram of the British Isles

Before we turn to the periodograms computed from the entire data stretches, we must consider one or two points of theory.

(a) If the periods are accidents, the peaks found in the periodograms from halves of the data should have  $H$  as high as from the whole stretch. If they are real and approximately constant, the  $H$ 's should increase in height and if they are more than slightly variable the  $H$ 's should decrease very much.

(b) The "spectroscope," for the periodogram may well be called such, has four times the resolving power when applied to these double data stretches. Where, previously, peaks merged or displaced each other, they may now be separated. Magnitudes and positions will be determined more accurately. Low peaks, which were comparable with the accidental errors, will begin to appear in their true form.

From the first half of the data of the Punjab the highest peaks are  $H=7.48$ , 5.40, and 4.42. From the latter half they are  $H=6.64$ , 5.77, and 4.73. From the total stretch they are  $H=7.98$ , 7.08, 6.23, and 6.01, a very substantial and consistent gain. The expectancy ratio of the highest of these is one in 2,900; that of the lowest of the four is one in 410. The first half of the Pacific Coast data gives  $H=5.90$ , 5.84, and 5.20, the latter half gives  $H=10.50$ , 5.03, and 3.79. The  $H$ 's from all of the data

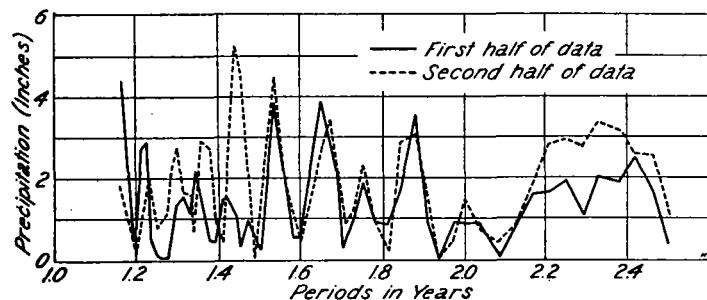


FIG. 4.—Rainfall periodogram of the Punjab (entire data series)

are 10.60, 8.55, 8.18, and 6.13. It is true that the gain in the case of the highest peak is almost negligible, but the consistency of the other gains makes our evidence just as definite as it was before. The expectancy ratios for these four highest peaks vary from one in 460 to one in 40,000.

Naturally, from what the halves disclosed, the British Isles data should show by far the best evidence. The first half of the data gives  $H=4.43$ , 3.90, and 3.86; the latter half  $H=5.33$ , 4.53, and 3.40. The total stretch given  $H=8.35$ , 6.18, 5.54, and 5.19, a truly remarkable gain in height. The expectancy ratios vary from one in 175 to one in 4,200.

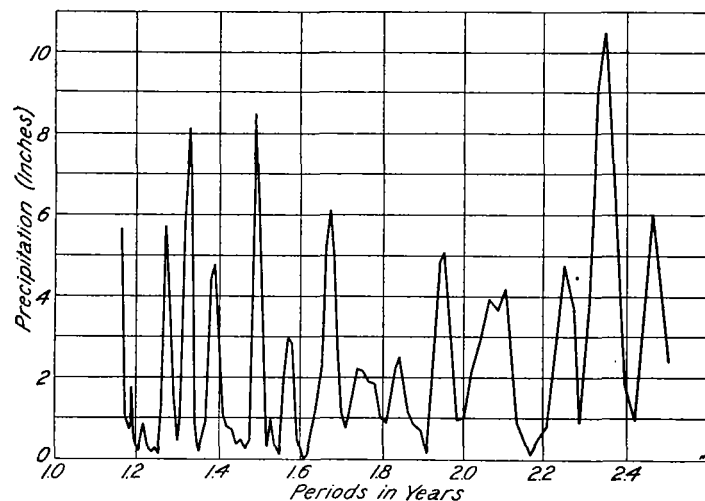


FIG. 5.—Rainfall periodogram of the Pacific coast (entire data series)

We have in previous papers considered very carefully the relationship of rainfall periods to the sun-spot period and have seen a very marked bias, enough to make it possible to compute the sun-spot period with accuracy from the rainfall data. In this section of the work the distance between adjacent harmonics is so small that slight displacements of the peaks by the accidental errors will cause the relationship to disappear. There is, however, a method by which it is possible to weigh the evidence submitted by the data as a whole. If there be a bias, the correlation coefficient between stretches of data separated by the sun-spot period or a multiple of it should be larger than would be expected by accident. It must be remembered, however, that we are able only to compare after an exact number of datum intervals and that, therefore, the correlation will be lower in general than

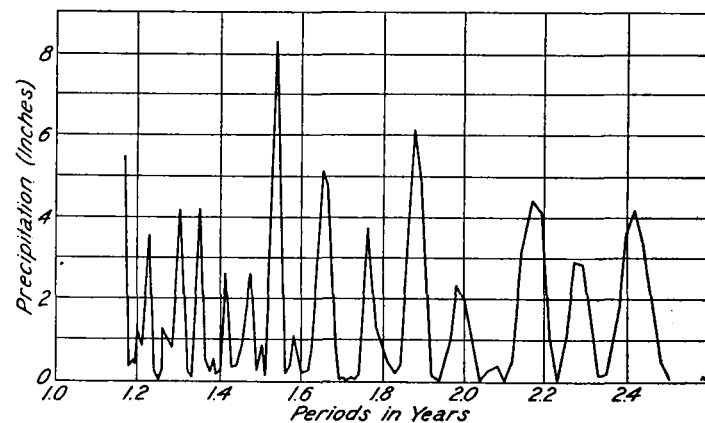


FIG. 6.—Rainfall periodogram of the British Isles (entire data series)

if we could match stretches more accurately. The Punjab shows the simplest relationship. Inspection shows a similarity after 47 phases or  $23\frac{1}{2}$  years. The correlation computed is  $r = +0.202$ ;  $\epsilon_r = 0.0737$ ;  $\frac{r}{\epsilon_r} = 2.75$ , which corresponds to an expectancy ratio of one to 243.

In the case of the other two sections we find that the correlation occurs after double the sun-spot period. For the Pacific coast the correlation is best after 89 phases, or  $44\frac{1}{2}$  years. It is

$$r = +0.194; \epsilon_r = 0.0860; \frac{r}{\epsilon_r} = 2.25$$

The expectancy ratio of such is 1 in 90. The period is exactly twice the sun-spot period.

The British Isles are much more exact and will be discussed in detail in the next issue of this journal. The best correlation occurs after 43 years.

$$r = +0.273; \epsilon_r = 0.0780; \frac{r}{\epsilon_r} = 3.50$$

The expectancy ratio is 1 in 1,100. The positions of the periodogram peaks all fall very close to harmonics of a period of about 43.1 years. Half this period, or 21.55 years, differs from the sun-spot period by only 0.7 year. The mean of the three sections gives 22.43 years as the sun-spot period.

In conclusion we find that, just as in the previous papers, when we consider shorter periods and therefore have more cycles available, the evidence becomes more definite in favor of:

(a) The real physical existence of a "spectrum" of rainfall periods.

(b) The relationship of rainfall periods to the sun-spot period.

(c) The approximate constancy of rainfall periods, in length at least.

Evidence against constancy of their magnitudes is lacking.

Beyond this, all is more or less speculation; and while it seems that the time has come for some sufficiently guarded speculation, the consideration of economic value clearly belongs in a separate paper.

The research committee of the Graduate School of the University of Kansas have assisted in this work by a grant through which computers were engaged to do more than half the routine computing. The Chief of the United States Weather Bureau and other officials there have very kindly made data available, even to the extent of making manuscript copies where necessary. As in all these papers, Professor Kester has encouraged and advised, contributing greatly to whatever success the work may find.

TABLE 1.—Percentage departures from normal precipitation, by half years,<sup>1</sup> for selected regions, 1834–1924

Year	Pacific coast	British isles A	The Punjab	British isles B	Year	Pacific coast	British isles A	The Punjab	British isles B
1834				-15	1836				+21
1835				-11					+23
				+4	1837				-12
				-1					-9

<sup>1</sup> The upper value of each section, given opposite each year, is the percentage of normal of that section for the first half of the year; the lower value for the second half of the year.

TABLE 1.—Percentage departures from normal precipitation, by half years, for selected regions, 1834–1924—Continued

Year	Pacific coast	British isles A	The Punjab	British isles B	Year	Pacific coast	British isles A	The Punjab	British isles B
1838				+6	1882	-11	+14	-27	+14
1839				-2		-19	+16	+10	+20
				-7	1883	-5	-6	-16	-3
1840				+28		-37	-3	-20	-1
				-12	1884	+87	-6	-21	-4
1841				-7		+45	-16	+12	-11
				+13	1885	-54	-3	+40	-3
1842				+32		+54	-21	-15	-19
				-21	1886	+20	+10	+53	+6
1843				-14		-47	+4	-2	+2
				+11	1887	-2	-38	-62	-37
1844				-11		-27	-16	+22	-16
				-9	1888	-11	-9	-36	-9
1845				-21		+20	+6	+5	+3
				-9	1889	-20	-11	+16	-12
1846				+3		+134	-10	-11	-11
				+9	1890	+17	-5	+32	-2
1847				+9		-40	-4	+26	+1
				-24	1891	-11	-30	+1	-30
1848				-14		-23	+21	-21	+22
				+42	1892	-16	-16	-59	-13
1849				+9		+34	-3	+47	+1
				+3	1893	+2	-35	+92	-32
1850				-9		-7	-2	+23	-2
				-9	1894	-5	+22	+92	+22
1851				-18		+28	-1	+34	-4
				+12	1895	+13	-32	+43	-33
1852	-30			-35		-37	+12	-33	+11
	+138			+6	1896	+14	-20	-30	-18
1853	+3			+12		+20	+13	-39	+5
	-22			+7	1897	-4	+22	-33	+17
1854	+8			-22		-33	-13	-11	-13
	-22			-32	1898	-50	-7	-6	-7
1855	+23			-30		-43	-12	-15	-12
	-28			-4	1899	-5	+14	-38	+15
1856	-4			+16		+28	-10	-70	-7
	-25			-10	1900	-24	+9	-36	+8
1857	-8			-15		-5	+9	+38	+12
	+20			-6	1901	-8	-4	+7	-9
1858	+15			-20		-21	-9	-13	-9
	+24			-18	1902	+4	-7	-24	-9
1859	-18			-11		+8	-21	-10	-17
	+26			+9	1903	-5	+33	-39	+30
1860	-1			+34		-36	+27	+26	+28
	+16			+25	1904	+9	+12	+22	+13
1861	-5			-17		+14	-20	-14	-18
	+68			+4	1905	+19	-3	-4	-3
1862	+77			+29		-43	-18	-11	-17
	-54			-5	1906	+48	+15	+35	+15
1863	-19			-1		-30	-4	+25	-2
	-39			-18	1907	+32	+14	+51	+14
1864	-44			-14		-13	-4	-29	-3
	+44			-18	1908	-23	+3	-20	+4
1865	-27			-2		-38	-9	+86	-6
	-4			+5	1909	+44	+3	+15	+4
1866	+32			+20		+46	+5	+40	+2
	+66			+11	1910	-29	+18	-13	+17
1867	+31			+17		-37	-4	+17	-4
	+108			-12	1911	+36	-10	+79	-6
1868	-23			-2		-51	-6	-43	-3
	-20			+11	1912	-1	+10	-16	+13
1869	+9			+12		-40	+7	-1	+7
	-24			-11	1913	-30	+14	+36	+16
1870	-24			-32		+15	-21	-4	-22
	-46			-9	1914	+20	+1	+32	+2
1871	-25			+4		-19	+3	+65	+3
	+63			-1	1915	+26	+3	+10	+5
1872	-8			+31		-7	+16	-45	+10
	-7			+28	1916	+38	+31	-36	+29
1873	-30			-14		0	+11	+45	+10
	+35			0	1917	-22	-9	+18	-8
1874	+7			-25		-71	+2	+127	+3
	-12			+2	1918	-3	-12	-10	-9
1875	-9			-17		+8	+26	-52	+27
	+50			+15	1919	0	+6	-14	+5
1876	+21			0		-41	-7	+17	-5
	-49			+16	1920	-31	+16	-12	+25
1877	-43			+33		+15	-10	-40	-5
	-24			+15	1921	-4	-24	-75	-16
1878	+73			+11		+37	-13	+23	-6
	-50			-10	1922	-12	-3	-36	+5
1879	+29			+24		+31	-11	+5	-11
	+22			-4	1923	-33	+1	+7	+5
1880	+23			-9		-49	+15	+23	+22
	+49			+25	1924	-49	+2	-14	+2
1881	+6			-3		0	+21	+30	+22
	-38			+10					

TABLE 2.—Rainfall periodograms for  $2\frac{1}{2}$  to  $1\frac{1}{6}$  years

Mean I. ....		Pacific coast								British Isles								The Punjab							
		Old				New				Old				New				Old				New			
		811.0				527.3				163.3				110.5				547.0				779.1			
P	F	I	H	H'	A	I	H	H'	A	I	H	H'	A	I	H	H'	A	I	H	H'	A	I	H	H'	A
2½	1.142	251	0.31	0.35	0.56	3,091	5.86	6.70	2.59	68	0.42	0.48	0.69	143	1.29	1.47	1.21	737	1.35	1.54	1.24	384	0.49	0.66	0.75
2½	1.147	731	0.90	1.03	1.01	5,548	10.50	12.04	3.47	279	1.71	1.96	1.39	279	2.52	2.89	1.70	870	1.59	1.82	1.35	18	0.02	0.02	0.14
2½	1.153	1,625	2.00	2.31	1.52	5,504	10.44	12.05	3.47	406	2.49	2.87	1.70	285	2.58	2.98	1.73	666	1.22	1.41	1.19	417	0.54	0.62	0.79
2½	1.159	2,949	3.64	4.21	2.05	5,223	9.90	11.47	3.38	312	1.91	2.21	1.49	353	3.20	3.70	1.92	416	0.76	0.88	0.94	1,558	2.00	2.32	1.52
2½	1.165	3,939	4.86	5.66	2.38	2,568	4.87	5.67	2.38	333	2.04	2.38	1.54	375	3.39	3.95	1.99	769	1.41	1.64	1.28	1,547	1.99	2.32	1.52
2½	1.172	3,416	4.21	4.94	2.22	679	1.29	1.51	1.23	179	1.10	1.29	1.14	306	2.77	3.25	1.80	1,200	2.19	2.57	1.60	2,470	3.17	3.72	1.93
2½	1.179	2,272	2.80	3.30	1.82	995	1.89	2.23	1.49	320	1.96	2.31	1.52	324	2.93	3.45	1.86	775	1.42	1.67	1.29	2,144	2.75	3.24	1.80
2½	1.187	759	0.94	1.11	1.05	1,999	3.79	4.50	2.12	267	1.64	1.95	1.39	300	2.72	3.23	1.80	79	0.14	0.17	0.41	2,759	3.54	4.20	2.05
2½	1.195	609	0.75	0.90	0.95	1,815	3.44	4.11	2.03	255	1.56	1.86	1.37	192	1.74	2.08	1.44	779	1.52	1.82	1.35	3,380	4.34	5.19	2.28
2½	1.204	2,670	3.29	3.97	1.99	626	1.19	1.43	1.19	136	0.83	1.00	1.00	98	0.89	1.07	1.03	1,214	2.22	2.68	1.64	3,817	4.90	5.90	2.43
2½	1.213	4,220	5.20	6.32	2.51	47	0.09	0.11	0.33	20	0.12	0.15	0.39	52	0.47	0.57	0.76	1,092	2.00	2.43	1.56	4,496	5.77	7.00	2.65
2½	1.223	2,929	3.61	4.42	2.10	110	0.21	0.26	0.51	133	0.81	0.99	0.99	77	0.70	0.86	0.93	131	0.24	0.29	0.54	3,050	3.92	4.80	2.19
2½	1.233	2,721	3.36	4.15	2.04	607	1.15	1.42	1.19	130	0.80	0.99	0.99	168	1.52	1.88	1.37	345	0.63	0.78	0.88	1,320	1.70	2.10	1.45
2½	1.242	3,569	4.40	5.47	2.33	986	1.87	2.33	1.52	138	0.85	1.06	1.03	60	0.54	0.67	0.82	1,144	2.09	2.60	1.61	633	0.81	1.01	1.00
2½	1.252	2,534	3.12	3.91	1.98	941	1.79	2.24	1.50	14	0.09	0.11	0.33	1	0.01	0.01	0.10	1,327	2.43	3.04	1.75	392	0.50	0.63	0.80
1½	1.261	931	1.15	1.45	1.20	219	0.42	0.53	0.73	150	0.92	1.16	1.08	156	1.41	1.78	1.34	787	1.44	1.82	1.35	753	0.97	1.22	1.11
1½	1.271	1,301	1.61	2.05	1.43	109	0.21	0.27	0.52	585	3.58	4.56	2.14	341	3.09	3.93	1.99	71	0.13	0.17	0.41	702	0.90	0.11	0.33
1½	1.280	1,597	1.97	2.52	1.59	894	1.70	2.18	1.48	269	1.65	2.11	1.45	317	2.87	3.68	1.92	51	0.09	0.12	0.35	494	0.60	0.08	0.28
1½	1.291	118	0.15	0.19	0.44	815	1.54	1.99	1.41	140	0.86	1.11	1.05	28	0.25	0.32	0.57	140	0.26	0.34	0.58	1,058	1.36	1.76	1.33
1½	1.304	861	1.06	1.38	1.17	290	0.55	0.72	0.85	147	0.90	1.17	1.08	100	0.91	1.19	1.09	229	0.42	0.50	0.71	2,809	3.61	4.71	2.17
1½	1.318	2,812	3.47	4.58	2.14	8	0.02	0.03	0.17	314	1.92	2.53	1.59	261	2.36	3.11	1.77	1,439	2.63	3.47	1.86	5,165	6.64	8.75	2.96
1½	1.327	1,551	1.91	2.54	1.59	107	0.20	0.27	0.52	160	0.98	1.30	1.14	146	1.32	1.75	1.32	2,419	4.42	5.87	2.42	3,385	4.35	5.78	2.40
1½	1.336	152	0.19	0.25	0.60	613	1.16	1.55	1.24	62	0.38	0.51	0.71	95	0.86	1.15	1.07	2,116	3.57	5.17	2.28	2,851	3.66	4.89	2.21
1½	1.346	1,649	2.03	2.73	1.65	1,201	2.28	3.07	1.75	364	2.23	3.00	1.73	242	2.19	2.95	1.72	1,380	2.52	3.39	1.84	975	1.25	1.68	1.30
1½	1.356	2,751	3.39	4.60	2.14	1,966	3.73	5.06	2.25	523	3.20	4.34	2.05	375	3.40	4.61	2.15	489	0.89	1.21	1.10	219	0.11	0.15	0.39
1½	1.367	2,416	2.98	4.07	2.02	2,648	5.03	6.88	2.62	637	3.90	5.33	2.31	314	2.84	3.88	1.97	18	0.03	0.04	0.20	10	0.01	0.01	0.10
1½	1.379	1,632	1.99	2.74	1.65	2,416	4.58	6.32	2.51	287	1.76	2.43	1.56	145	1.31	1.80	1.34	38	0.07	0.10	0.32	301	0.39	0.54	0.74
1½	1.391	2,148	2.65	3.69	1.92	1,907	3.62	5.04	2.24	85	0.52	0.72	0.85	49	0.44	0.61	0.78	433	0.79	1.10	1.05	846	1.09	1.52	1.23
1½	1.404	2,702	3.33	4.68	2.16	1,353	2.57	3.61	1.90	86	0.53	0.74	0.86	143	1.29	1.81	1.35	1,195	2.19	3.08	1.76	1,237	1.58	2.22	1.49
1½	1.417	1,484	1.83	2.59	1.61	250	0.47	0.67	0.82	340	2.08	2.95	1.99	217	1.97	2.79	1.67	2,519	4.60	6.52	2.56	1,429	1.84	2.61	1.62
1½	1.431	141	0.17	0.24	0.49	88	0.17	0.24	0.49	630	3.86	5.53	2.36	501	4.53	6.49	2.55	3,513	6.43	9.20	3.64	1,619	2.08	2.98	1.73
1½	1.446	3,010	3.71	5.36	2.31	1,444	2.74	3.96	1.99	330	2.02	2.92	1.71	301	2.72	3.93	1.98	4,085	7.48	10.81	3.29	1,583	2.04	2.95	1.72
1½	1.462	3,121	3.85	5.64	2.37	1,600	3.04	4.45	2.11	45	0.28	0.41	0.64	98	0.89	1.30	1.14	3,920	7.17	10.50	3.24	1,301	1.67	2.45	1.57
1½	1.475	3,947	4.87	7.19	2.68	860	1.63	2.40	1.55	98	0.60	0.89	0.94	4	0.04	0.06	0.24	3,709	6.79	10.01	3.17	865	1.11	1.64	1.28
1½	1.488	1,064	1.34	1.99	1.41	338	0.64	0.95	0.97	159	0.97	1.44	1.20	260	2.35	3.50	1.87	2,961	5.42	8.07	2.84	214	0.27	0.40	0.63
1½	1.501	86	0.11	0.17	0.41	282	0.53	0.80	0.89	63	0.39	0.59	0.77	511	4.62	6.94	2.64	1,666	3.05	4.59	2.14	33	0.04	0.06	0.25
1½	1.515	231	0.28	0.42	0.65	1,031	1.96	2.97	1.72	180	1.10	1.67	1.29	589	5.33	8.08	2.84	768	1.40	2.12	1.46	34	0.04	0.06	0.25
1½	1.529	318	0.39	0.60	0.77	1,742	3.31	5.06	2.25	261	1.60	2.45	1.57	193	1.75	2.68	1.64	161	0.29	0.44	0.66	128	0.16	0.24	0.49
1½	1.544	913	1.13	1.75	1.32	1,528	2.90	4.48	2.11	240	1.47	2.27	1.51	50	0.45	0.70	0.84	243	0.44	0.68	0.83	300	0.39	0.60	0.77
1½	1.560	2,036	2.51	3.92	2.98	1,360	2.58	4.03	2.01	72	0.44	0.69	0.83	106	0.96	1.50	1.23	392	0.72	1.13	1.06	381	0.49	0.76	0.87
1½	1.577	2,662	3.28	5.17	2.27	269	0.51	0.80	0.89	87	0.53	0.84	0.92	302	2.73	4.31	2.08	392	0.72	1.13	1.06	585	0.75	1.18	1.09
1½	1.594	943	1.16	1.85	1.36	30	0.06	0.09	0.30	244	1.49	2.38	1.54	322	2.91	4.65	2.16	694	1.27	2.03	1.43	1,288	1.65	2.63	1.62
1½	1.612	2,534	3.12	5.03	2.24	825	1.57	2.53	1.69	370	2.26	3.65	1.91	82	0.74	1.19	1.09	1,865	3.41	5.50	2.35	1,996	2.56	4.13	2.03
1½	1.631	4,742	5.84	9.54	3.08	1,363	2.59	4.23	2.06	188	1.15	1.88	1.37	160	1.45	2.37	1.54	2,727	4.99	8.15	2.86	2,636	3.38	5.52	2.35
1½	1.652	2,761	3.41	5.65	2.38	1,030	1.84	3.04	1.74	240	1.47	2.43	1.56	176	1.59	2.63	1.62	2,948	5.40	8.93	2.99	2,612	3.35	5.54	2.36
1½	1.674	587	0.72	1.21	1.10	26	0.05	0.08	0.28</																

TABLE 3.—Rainfall periodograms (entire data series) for  $2\frac{1}{4}$  to  $1\frac{1}{2}$  years

Pacific coast					British Isles				The Punjab				Pacific coast					British Isles				The Punjab											
Mean I...					664.5				136.0				636.9				Mean I...					664.5				136.0				636.9			
P	F	I	H	H'	A	I	H	H'	A	I	H	H'	A	P	F	I	H	H'	A	I	H	H'	A	I	H	H'	A						
1.142	1.579	2.38	2.72	1.65	9	0.07	0.08	0.28	666	1.05	1.20	1.10	1.141	2.006	3.02	4.26	2.06	59	0.43	0.61	0.78	2.660	4.18	5.89	2.43								
1.147	2.980	4.48	5.12	2.26	76	0.56	0.64	0.80	326	0.51	0.58	0.76	1.417	1.420	2.14	3.07	1.75	34	0.25	0.35	0.59	3.830	6.01	8.52	2.92								
1.147	3.981	5.99	6.86	2.62	265	1.95	2.24	1.50	488	0.77	0.88	0.94	1.424	1.07	0.16	0.23	0.48	513	3.77	5.37	2.32	3.670	5.76	8.21	2.87								
1.150	2.508	3.78	4.35	2.09	455	3.34	3.84	1.96	618	0.97	1.12	1.06	1.431	245	0.37	0.53	0.73	1,137	8.35	11.95	3.46	2,104	3.30	4.72	2.17								
1.153	623	0.94	1.08	1.04	572	4.21	4.86	2.20	619	0.97	1.12	1.06	1.438	650	0.98	1.41	1.19	748	5.50	7.91	2.81	709	1.11	1.60	1.26								
1.156	1.225	1.84	2.13	1.46	481	3.53	4.08	2.02	514	0.81	0.94	0.97	1.446	232	0.35	0.51	0.71	1,211	0.89	1.29	1.14	901	1.41	2.04	1.43								
1.159	4.973	7.49	8.68	2.95	236	1.74	2.02	1.42	378	0.59	0.68	0.82	1.454	880	1.33	1.93	1.39	28	0.21	0.31	0.56	2,565	4.03	5.86	2.42								
1.162	7.047	10.60	12.32	3.51	36	0.26	0.30	0.55	39	0.06	0.07	0.26	1.462	3,900	5.88	8.60	2.93	112	0.82	1.20	1.10	4,513	7.08	10.36	3.22								
1.165	5.940	8.94	10.41	3.23	28	0.21	0.24	0.49	135	0.21	0.24	0.49	1.469	5,675	8.55	12.56	3.54	43	0.32	0.47	0.67	4,315	6.78	9.96	3.16								
1.168	2.435	3.66	4.27	2.07	172	1.27	1.48	1.22	364	0.57	0.67	0.82	1.475	4,180	6.35	9.30	3.05	98	0.72	1.06	1.03	2,924	4.59	6.77	2.60								
1.172	606	0.91	1.07	1.03	388	2.85	3.34	1.83	2,002	3.14	3.68	1.92	1.481	1,871	2.82	4.18	2.04	272	2.06	2.96	1.72	1,705	2.67	3.95	1.99								
1.175	2.463	3.71	4.36	2.09	397	2.92	3.43	1.85	2,364	3.71	4.36	2.09	1.488	1,981	3.10	4.77	0.70	352	2.59	3.85	1.96	694	1.09	1.62	1.27								
1.179	3.161	4.76	5.61	2.37	144	1.06	1.25	1.12	2,347	3.69	4.35	2.09	1.494	154	0.23	0.34	0.58	261	1.92	2.87	1.69	930	1.46	2.18	1.48								
1.183	1.417	2.13	2.52	1.59	1	0.01	0.01	0.10	1,427	2.24	2.65	1.63	1.501	350	0.53	0.80	0.89	120	0.88	1.32	1.15	1,120	1.76	2.64	1.62								
1.187	532	0.80	0.95	0.97	145	1.07	1.27	1.13	889	1.39	1.65	1.28	1.508	267	0.46	0.60	0.77	78	0.56	0.84	0.92	802	1.26	1.90	1.38								
1.191	336	0.51	0.61	0.78	566	4.16	4.95	2.22	1,613	2.53	3.01	1.73	1.515	212	0.32	0.48	0.69	55	0.46	0.61	0.78	398	0.62	0.94	0.97								
1.195	106	0.16	0.19	0.44	602	4.42	5.23	2.29	2,490	3.91	4.67	2.16	1.522	416	0.63	0.96	0.98	50	0.37	0.56	0.75	122	0.19	0.29	0.54								
1.199	582	0.88	1.06	1.03	438	3.22	3.86	1.96	1,508	2.37	2.84	1.69	1.529	464	0.70	1.07	1.03	247	1.82	2.78	1.67	193	0.30	0.46	0.68								
1.204	1.921	2.89	3.48	1.87	83	0.61	0.74	0.80	311	0.49	0.59	0.77	1.536	694	1.05	1.61	1.27	355	2.61	4.01	2.00	227	0.36	0.55	0.74								
1.208	2.841	4.28	5.17	2.27	2	0.01	0.01	0.10	923	1.45	1.75	1.32	1.544	1,089	2.40	3.71	1.98	229	1.66	2.61	1.62	199	0.31	0.48	0.69								
1.213	2.370	3.57	4.33	2.08	65	0.48	0.58	0.76	2,617	4.13	5.01	2.24	1.552	2,236	3.37	5.23	2.29	41	0.30	0.47	0.69	4	0.01	0.02	0.14								
1.218	2.626	3.95	4.81	2.19	45	0.33	0.40	0.63	2,816	4.47	5.44	2.33	1.560	3,166	4.77	7.44	2.73	24	0.18	0.28	0.53	295	0.46	0.72	0.85								
1.223	1.877	2.82	3.45	1.86	1	0.01	0.01	0.10	1,697	2.66	3.25	1.80	1.568	2,998	4.52	7.08	2.66	82	0.60	0.94	0.97	696	1.09	1.71	1.31								
1.228	1.488	2.24	2.63	1.62	132	0.97	1.19	1.09	709	1.11	1.36	1.17	1.577	1,966	2.96	4.67	2.10	30	0.22	0.35	0.59	1,255	1.97	3.11	1.76								
1.233	660	0.99	1.22	1.10	264	1.94	2.39	1.55	1,059	1.66	2.05	1.43	1.585	602	0.91	1.44	1.26	76	0.56	0.89	0.94	1,520	2.39	3.79	1.95								
1.238	630	0.95	1.18	1.09	325	2.39	2.96	1.72	1,385	2.17	2.69	1.64	1.594	418	0.63	1.00	1.00	379	2.79	4.45	2.11	1,696	2.66	4.24	2.06								
1.242	1.767	2.66	3.30	1.82	136	1.06	1.24	1.11	1,180	1.85	2.30	1.52	1.603	1,72	0.26	0.42	0.65	576	4.24	6.86	2.61	886	1.39	2.23	1.40								
1.247	3.381	5.09	6.34	2.52	48	0.35	0.44	0.66	1,419	2.23	2.78	1.67	1.612	530	0.80	1.29	1.14	436	3.21	5.17	2.27	156	0.24	0.39	0.62								
1.252	3.220	4.85	6.07	2.46	1	0.01	0.01	0.10	983	1.54	1.93	1.39	1.621	2,990	4.50	7.30	2.70	226	1.66	2.63	1.64	164	0.26	0.42	0.65								
1.257	1.540	2.32	2.92	1.71	16	0.12	0.15	0.39	157	0.25	0.31	0.56	1.631	5,430	8.18	13.34	3.65	17	0.12	0.26	0.45	1,550	2.43	3.96	1.99								
1.261	106	0.16	0.20	0.45	192	1.41	1.78	1.33	105	0.16	0.20	0.45	1.641	5,025	7.57	12.42	3.52	50	0.37	0.61	0.78	3,830	6.01	9.80	3.14								
1.266	431	0.65	0.82	0.91	667	4.90	6.20	2.49	30	0.05	0.06	0.24	1.652	3,220	4.85	8.02	2.83	325	2.39	3.95	1.99	5,078	7.98	13.19	3.63								
1.271	566	0.85	1.08	1.04	841	6.18	7.85	2.80	129	0.20	0.25	0.50	1.663	643	0.97	1.61	1.27	578	4.25	7.07	2.66	4,380	6.88	11.44	3.38								
1.276	755	1.14	1.45	1.20	543	3.99	5.09	2.26	199	0.31	0.40	0.63	1.674	374	0.56	0.94	0.97	560	4.12	6.90	2.63	3,745	5.88	9.87	3.14								
1.280	1.717	2.58	3.30	1.82	53	0.39	0.50	0.71	173	0.27	0.35	0.59	1.685	955	1.44	2.42	1.56	214	1.57	2.65	1.63	835	1.31	2.21	1.49								
1.285	1.565	2.36	3.03	1.74	29	0.21	0.27	0.52	576	0.90	1.16	1.08	1.697	2,380	3.59	6.09	2.47	102	0.75	1.27	1.13	93	0.15	0.25	0.50								
1.291	557	0.84	1.08	1.04	63	0.46	0.59	0.77	782	1.23	1.59	1.26	1.709	3,770	5.68	9.71	3.12	146	1.07	1.83	1.35	56	0.09	0.15	0.39								
1.297	663	1.00	1.30	1.14	119	0.88	1.14	1.07	758	1.19	1.54	1.24	1.721	2,614	3.93	6.76	2.60	181	1.33	2.29	1.51	389	0.61	1.05	1.02								
1.304	1.233	1.86	2.43	1.56	186	1.37	1.79	1.34	2,067	3.15	4.11	2.03	1.733	836	1.26	2.18	1.48	49	0.36	0.62	0.79	411	0.65	1.13	1.06								
1.311	1.254	1.89	2.48	1.57	501	3.68	4.82	2.20	3,458	5.62	7.37	2.71	1.746	110	0.17	0.30	0.55	6	0.04	0.07	0.26	812	0.27	2.22	1.49								
1.318	1.519	2.29	3.02	1.74	152	1.12	1.48	1.22	3,984	6.23	8.21	2.87	1.754	154	0.23	0.40	0.63	37	0.27	0.47	0.69	1,043	1.64	2.88	1.70								
1.323	1.545	2.32	3.07	1.75	30	0.22	0.29	0.54	1,004	1.57	2.08	1.44	1.763	194	0.29	0.51	0.71	230	1.69	2.98	1.73	628	0.99	1.75	1.32								
1.327	1.217	1.83	2.43	1.56	7	0.05	0.07	0.26	359	0.56	0.74	0.86	1.772	147	0.22	0.39	0.62	432	3.17	5.62	2.37	433	0.68	1.20	1.10								
1.332	804	1.21	1.61	1.27	7	0.05	0.07	0.26	40	0.06	0.08	0.28	1.781	174	0.26	0.46	0.68	484	3.56	6.34	2.52	360	0.50	1.00	1.00								
1.336	439	0.66	0.88	0.94	6	0.04	0.05	0.22	708	1.11	1.48	1.22	1.790	261	0.39	0.70	0.84	342	2.51	4.49	2.12	644	1.01	1.81	1.35								
1.341	818	1.23	1.65	1.28	11	0.08	0.11	0.33																									